Assembly of Finite Element Methods on Graphics Processors.

Cris Cecka Adrian Lew Eric Darve

Department of Mechanical Engineering
Institute for Computational and Mathematical Engineering
Stanford University

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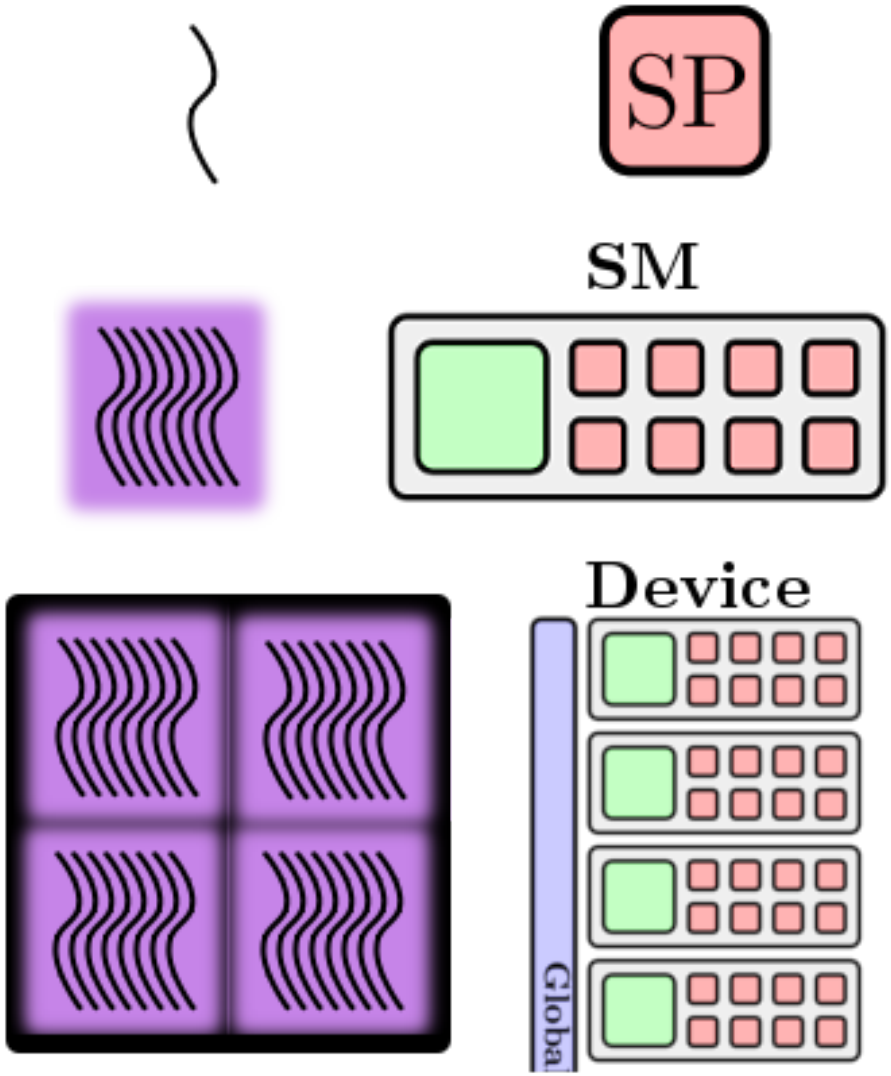
GPU Computing



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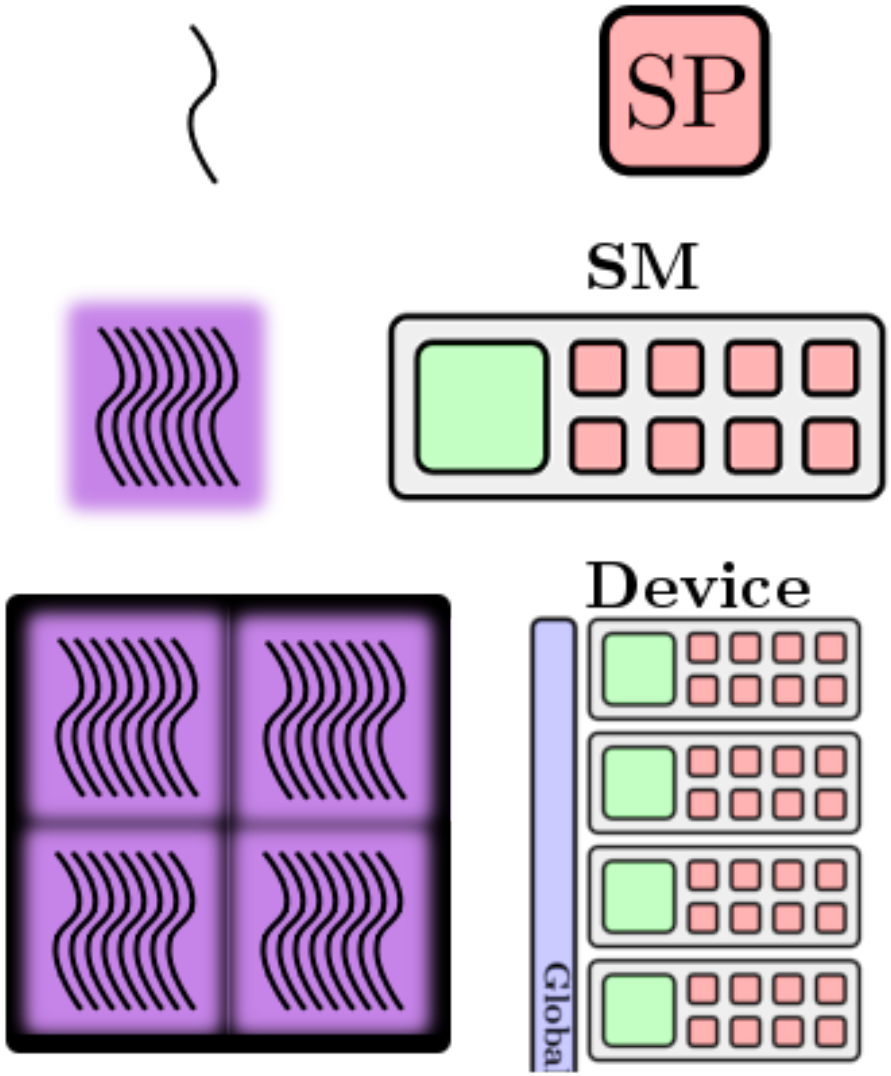

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  - On-Chip *Registers*
  - Off-chip *Local Memory*

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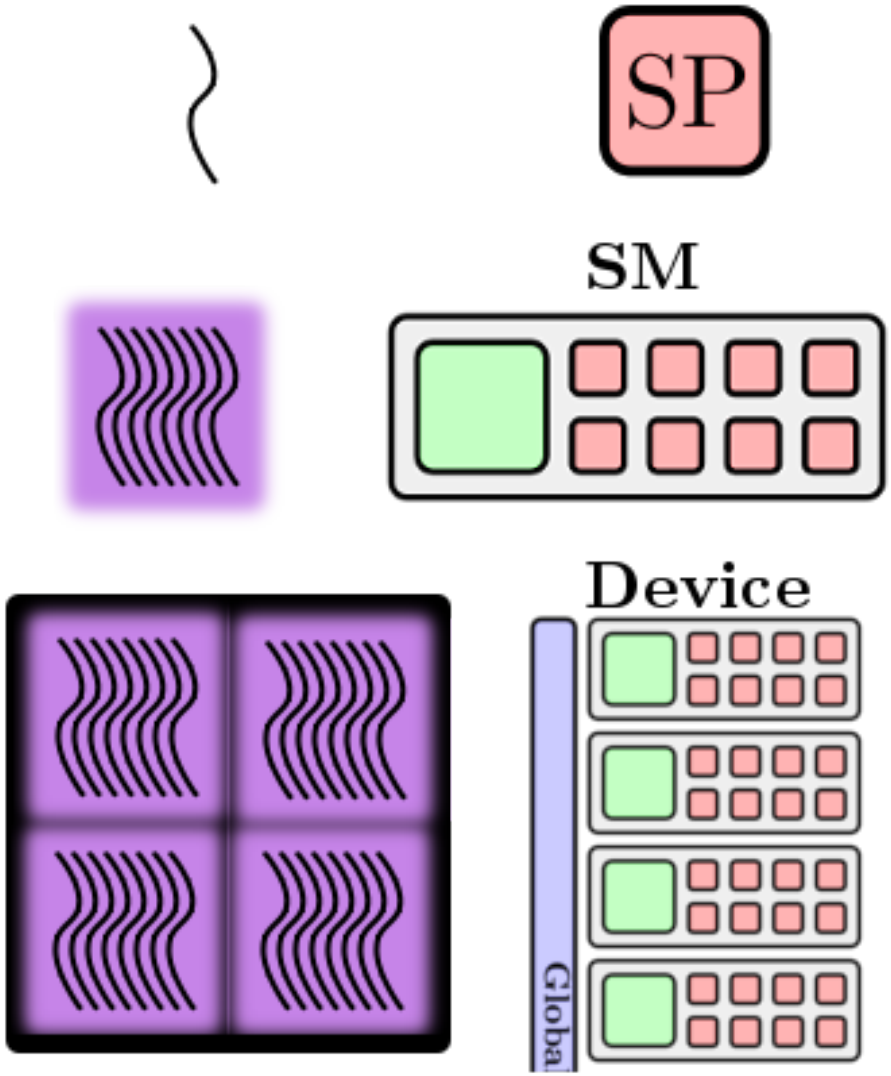

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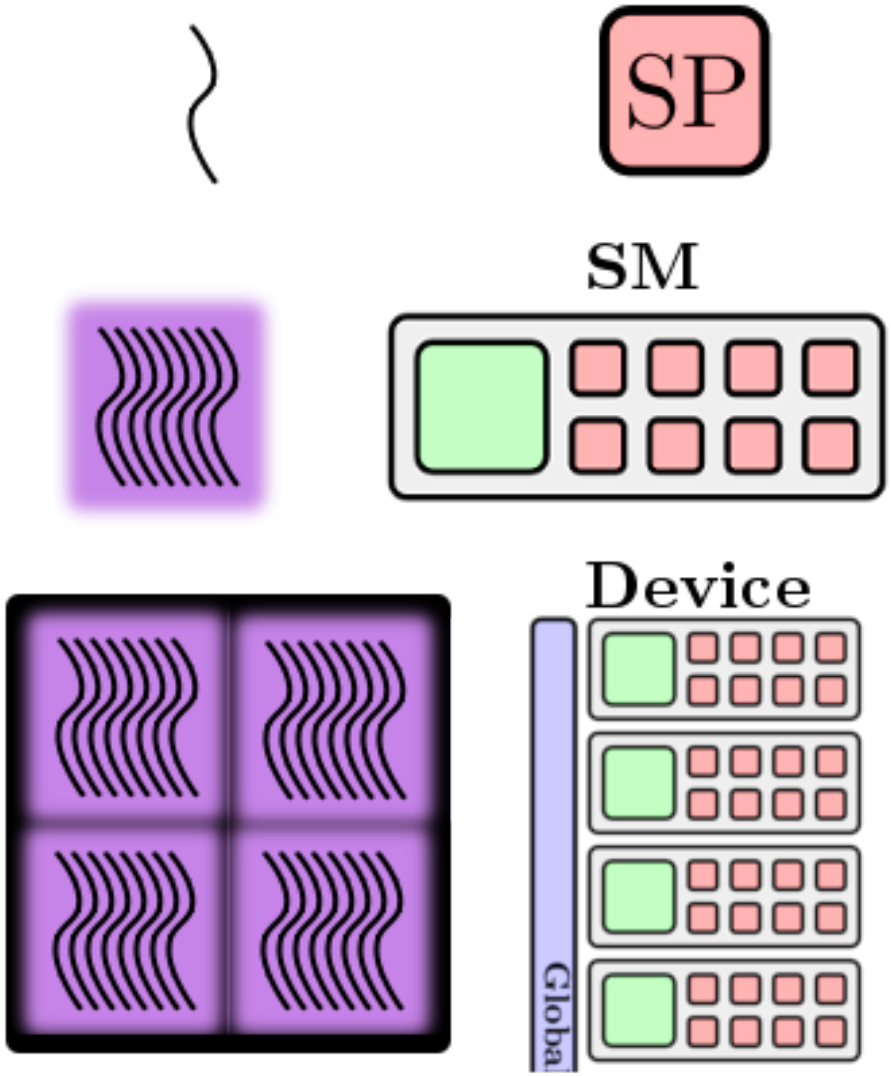

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  - On-chip *Shared Memory*
- *Grid* of blocks executed on the *device*
  - Off-chip *Global Memory*

Why FEM Assembly on the GPU?

- Complex, real-time physics common on the GPU.
  - Gaming and graphics community.
  - Simulation and visualization community.
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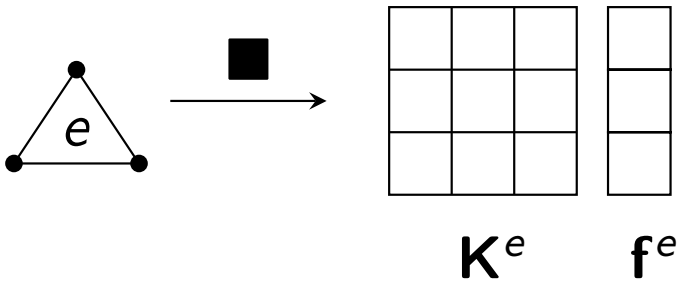
- Non-linear and time-dependent problems require many assembly procedures.
- Can assemble, solve, update, and visualize on the GPU
  - Completely avoid costly transfers with CPU.
  - Fast (real-time) simulations with visualization.

FEM Direct Assembly

Most common FEM assembly procedure:

- Compute element data.
  - One by one.



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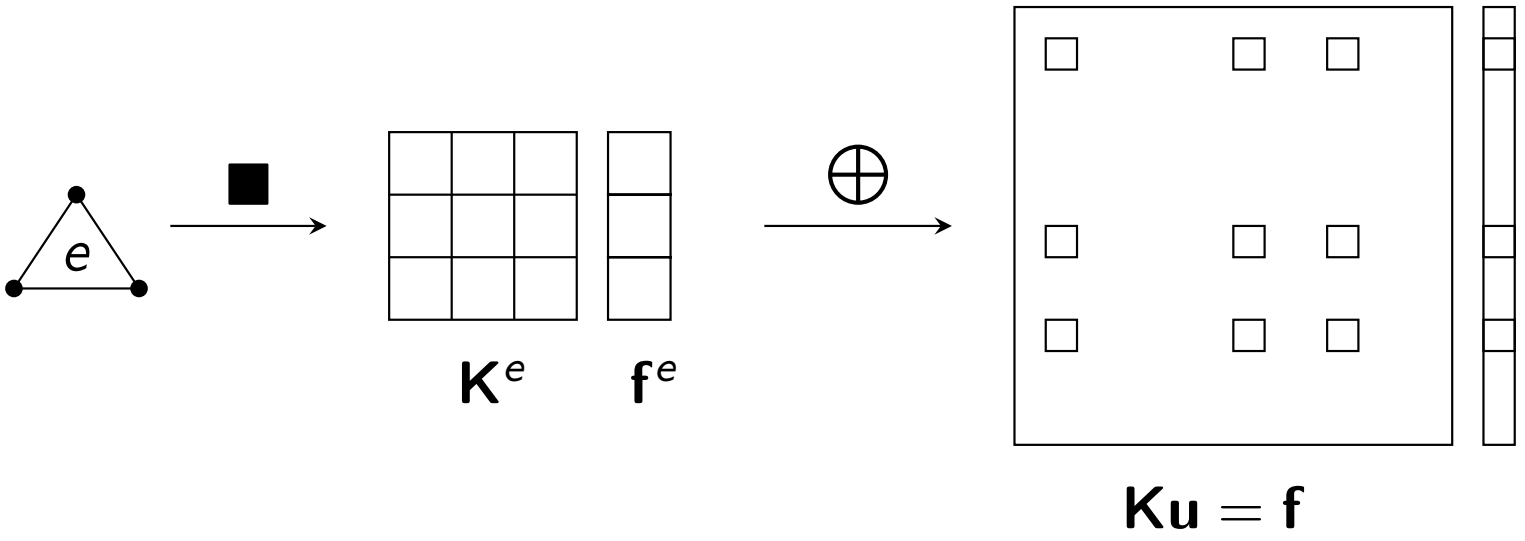
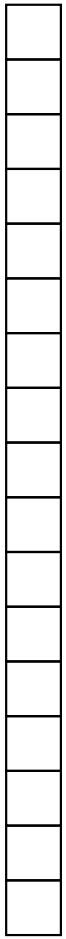
- Accumulate into global system.
  - Using a local index to global index mapping.



Data Flow

Nodal Data



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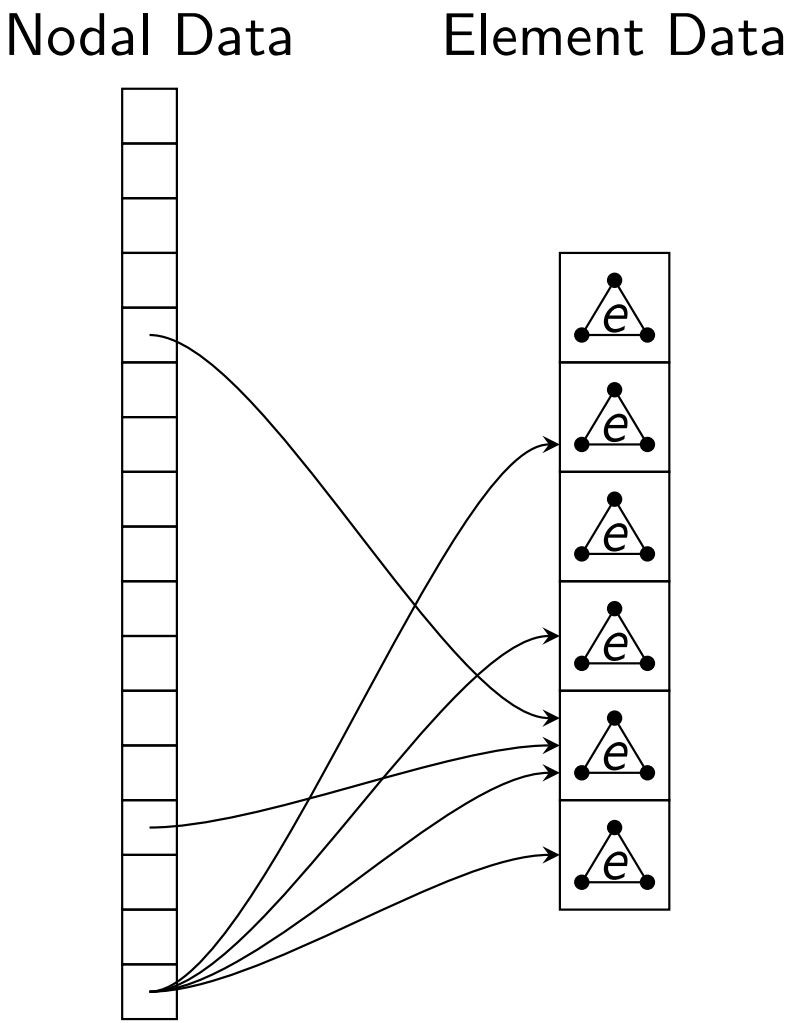

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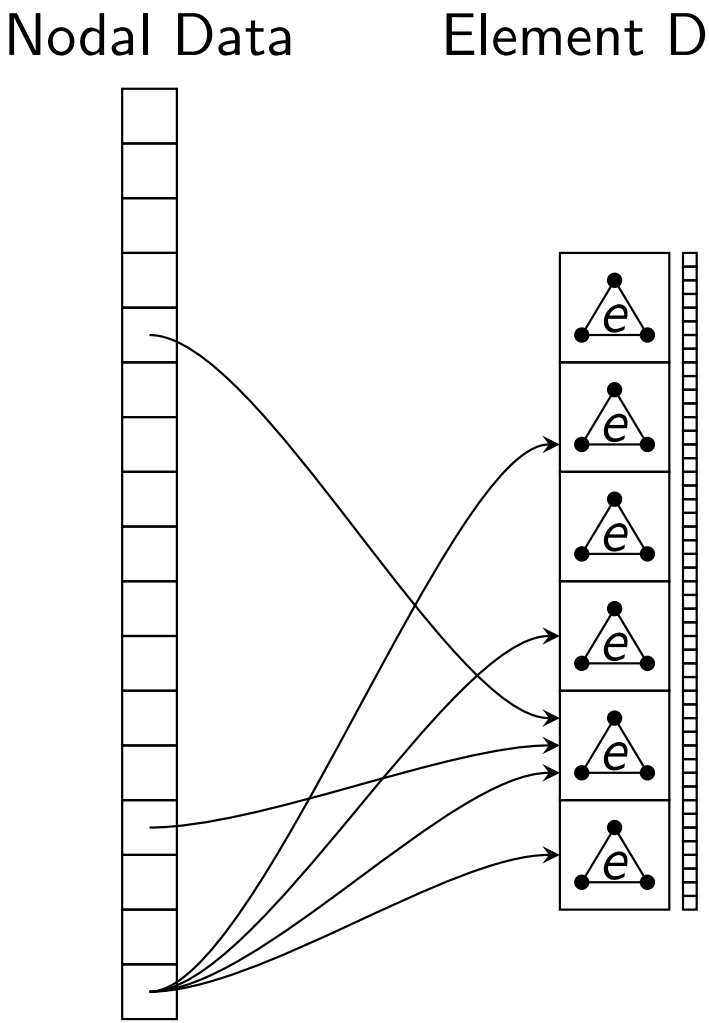

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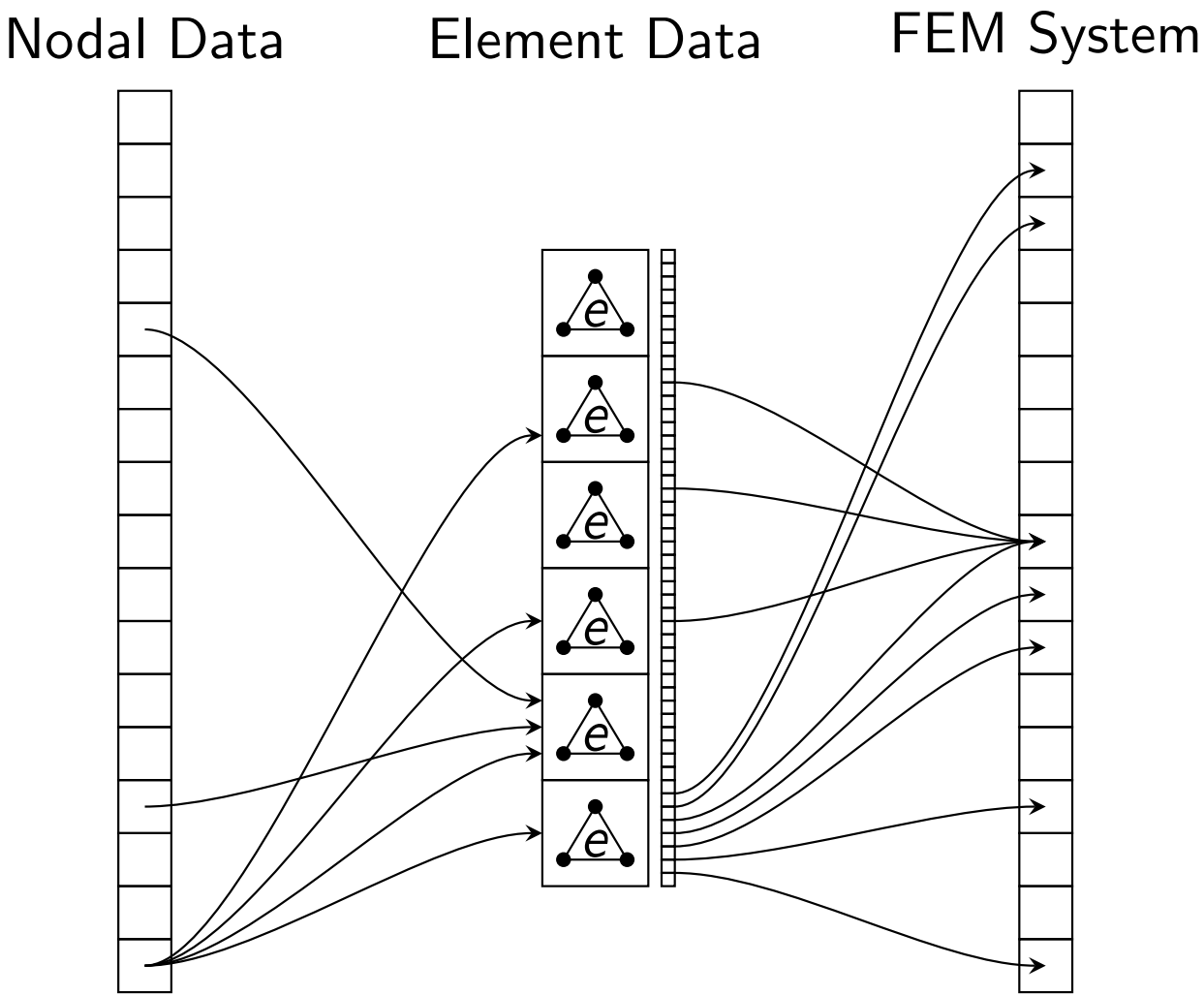

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- Distribute the task into independent blocks of work.
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- Maximize flop::word ratio.
  - Minimize global memory transactions.
  - Use exposed memory hierarchy to maximize data reuse.

GPU FEM Assembly Strategies

Two Key Choices:

Store Element Data In

Threads Assemble By

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  - Min computation.
  - SIMD.
  - Race conditions.

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- Transform into a vertex coloring problem.
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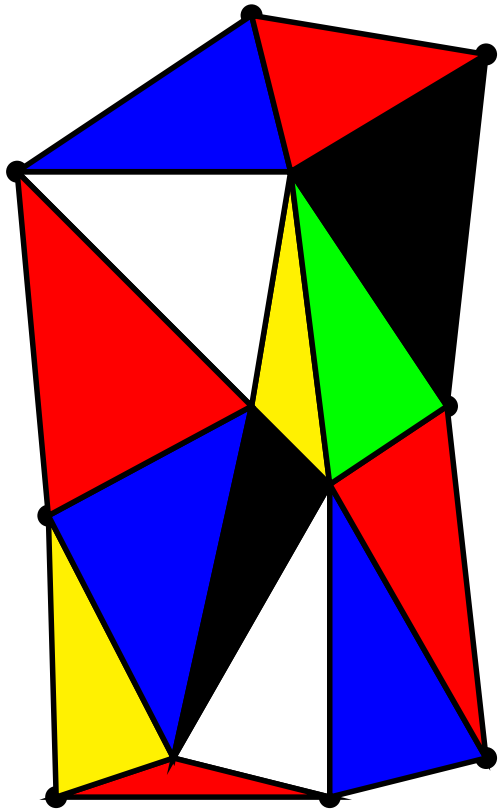

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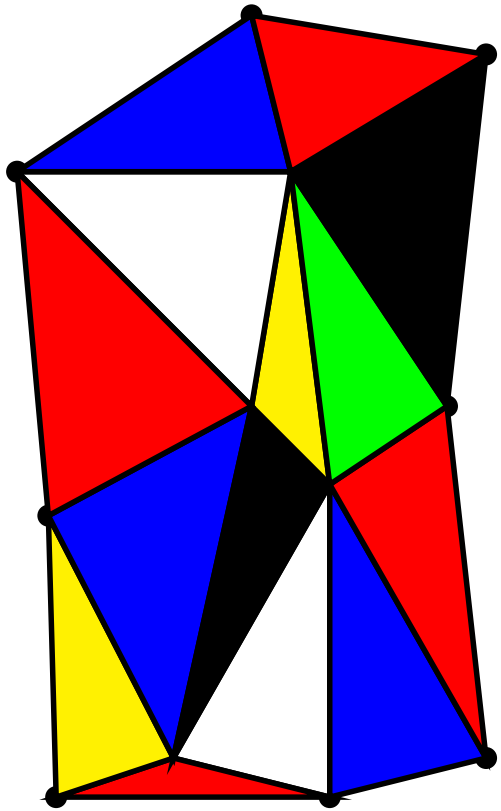
Problems.

- No sharing of nodal or element data.
- Little utilization of GPU resources.



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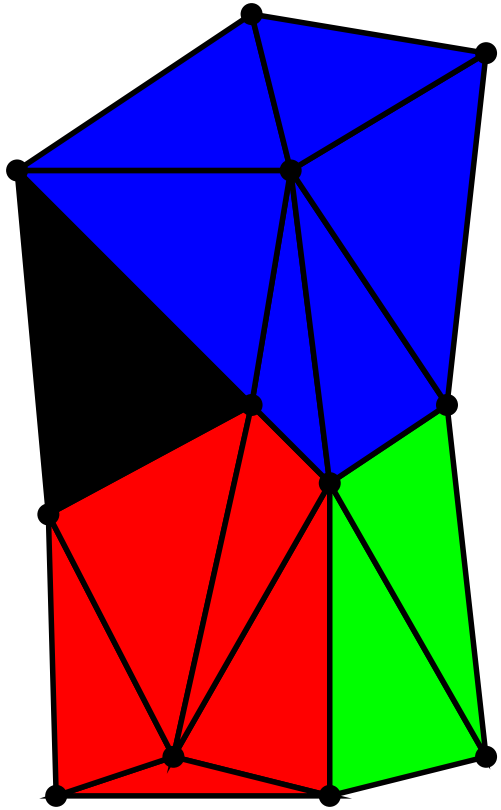
Optimizing:

- Cluster the elements so they share nodes.
- Prefetch nodal data into shared memory.
- Up to almost 3x speedup.



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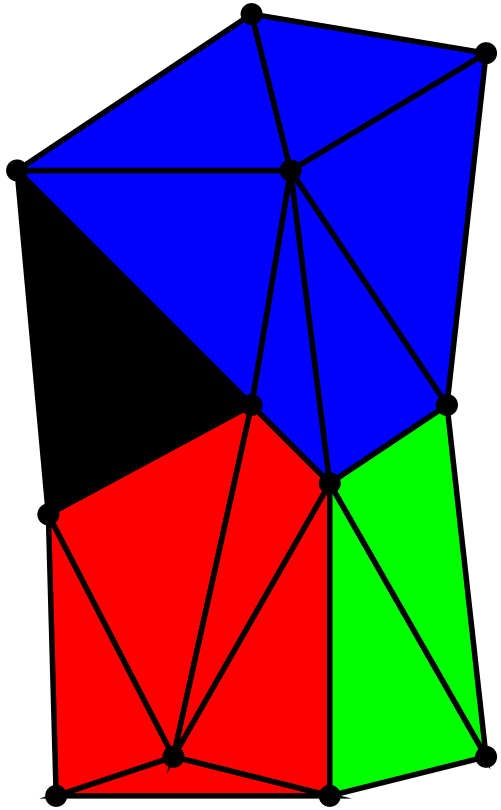
- Two passes through global memory.
- Limited by global memory size.



Global-NZ Data Flow

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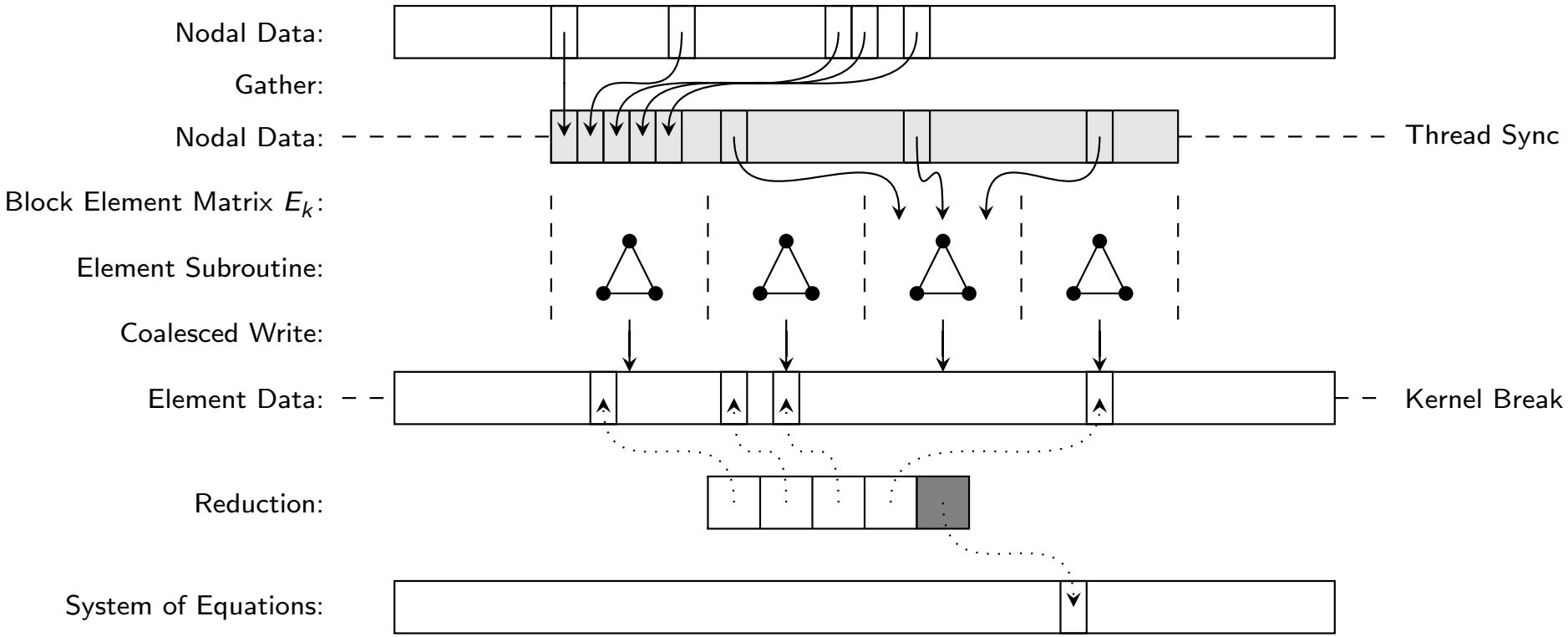

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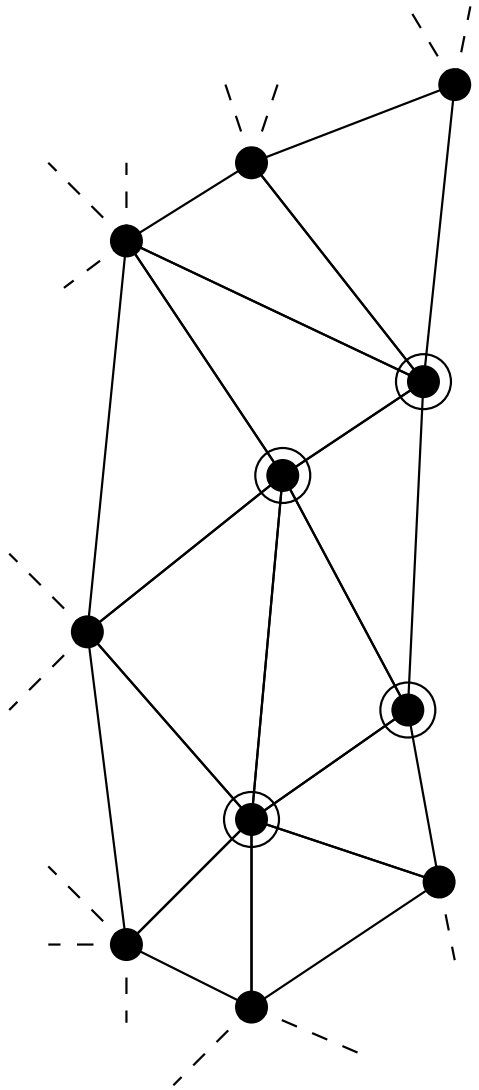

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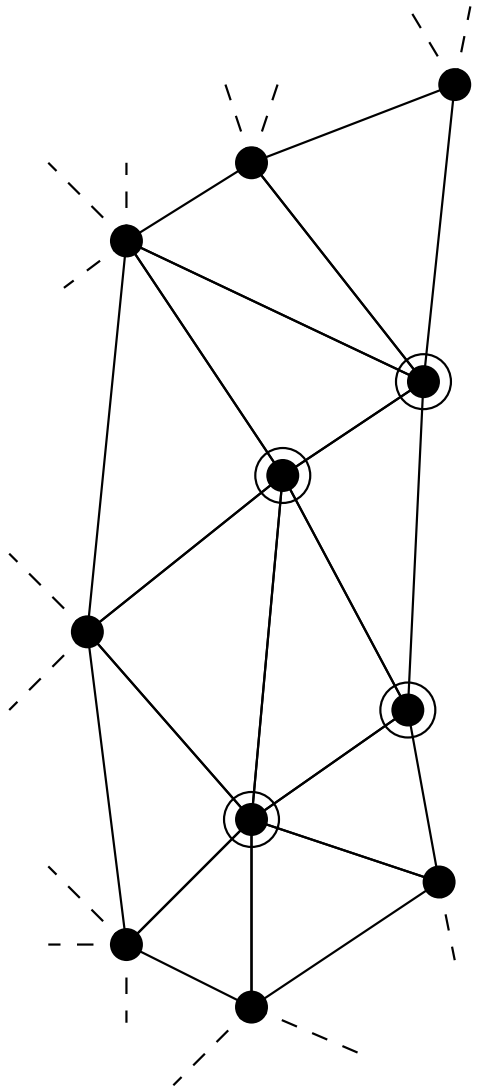
Problems.

- Shared memory size is very limiting.



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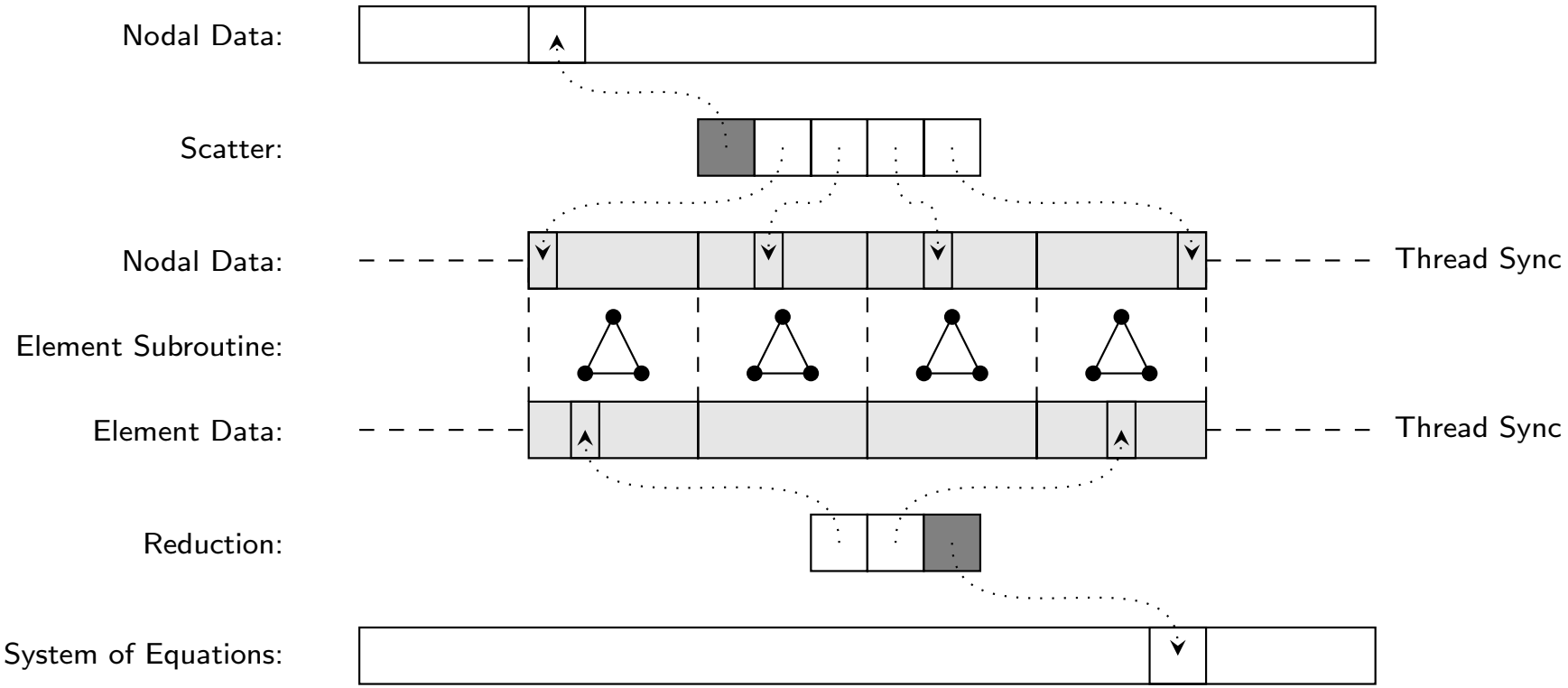

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General procedure:

- Make a set of operations to be done for each partition.
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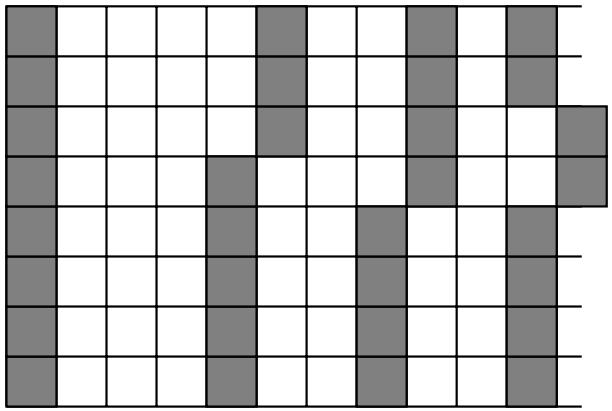

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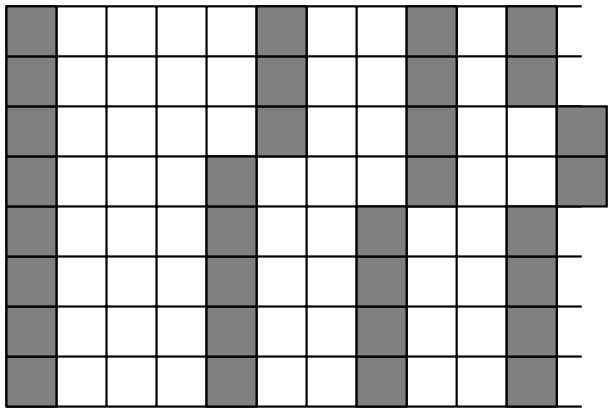

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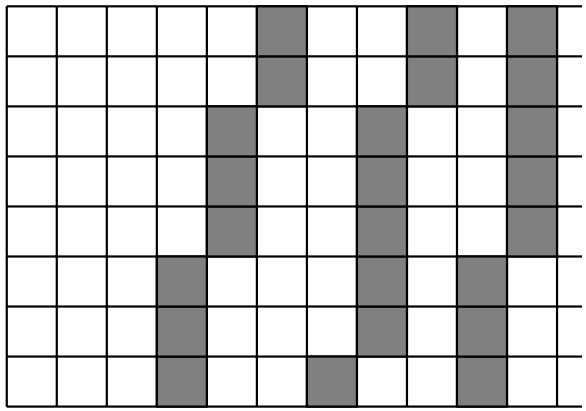

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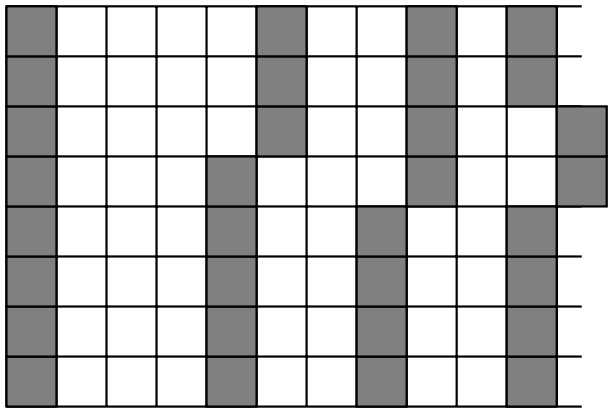

- Very fast.
- Highly adaptable.

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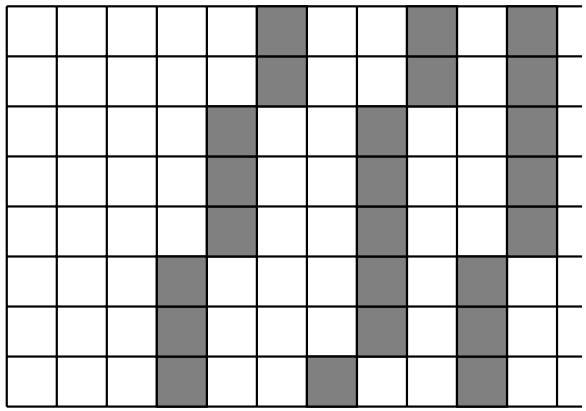
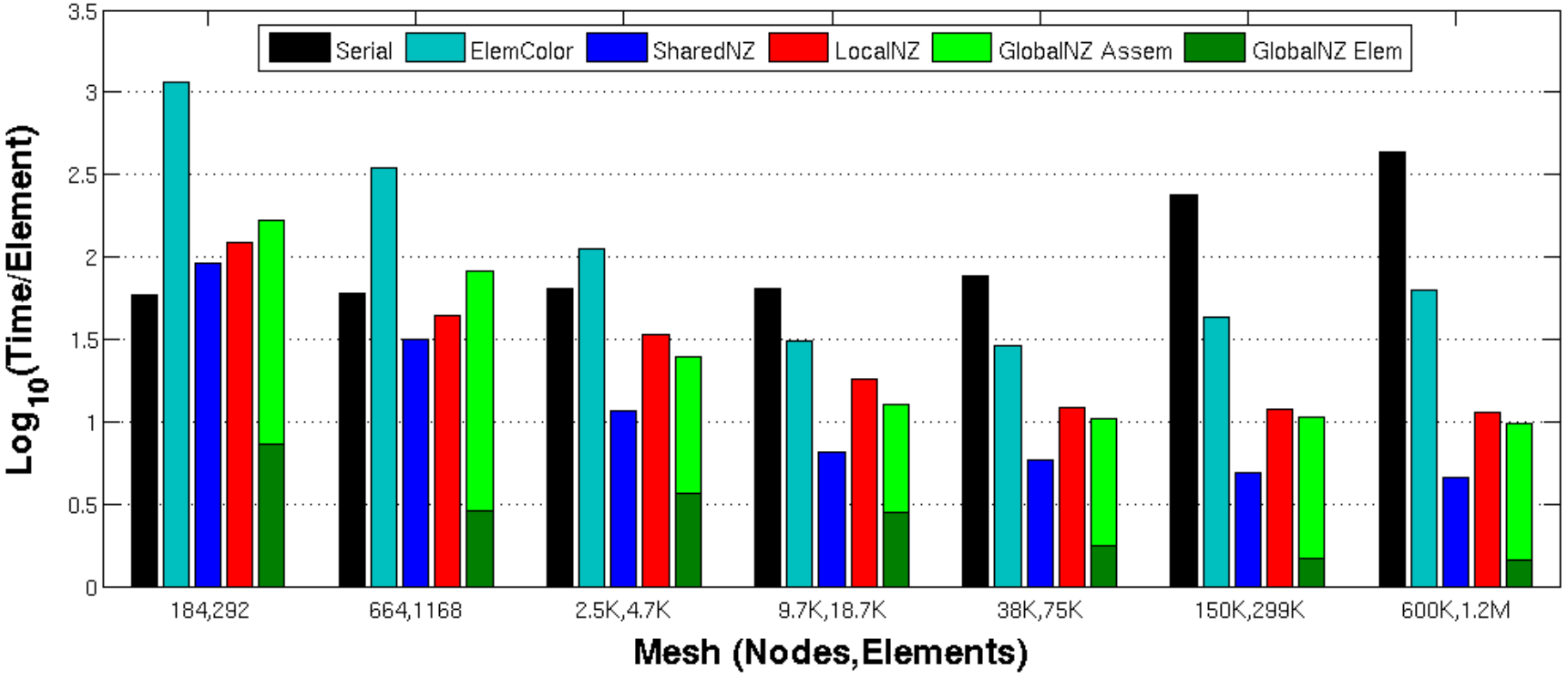
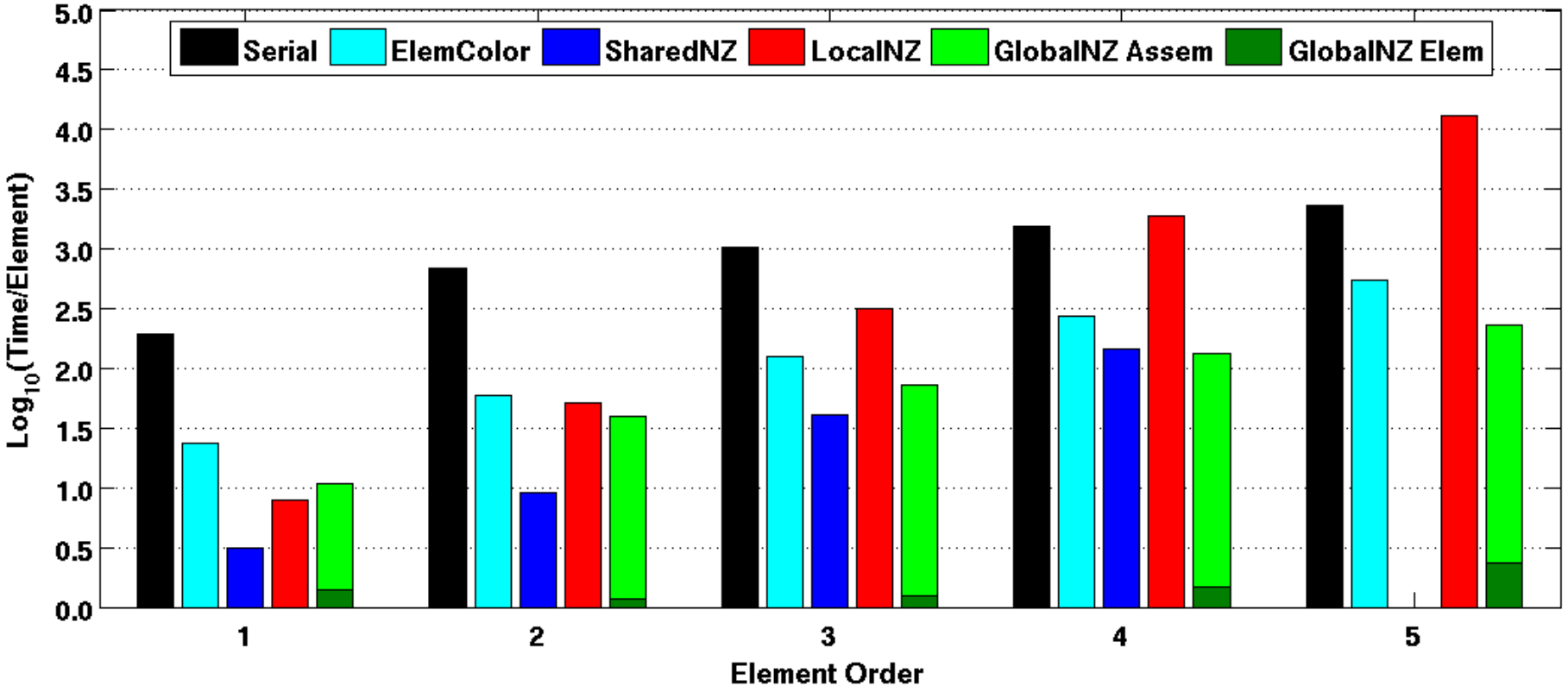


- Significant setup cost.
- Significant memory cost.

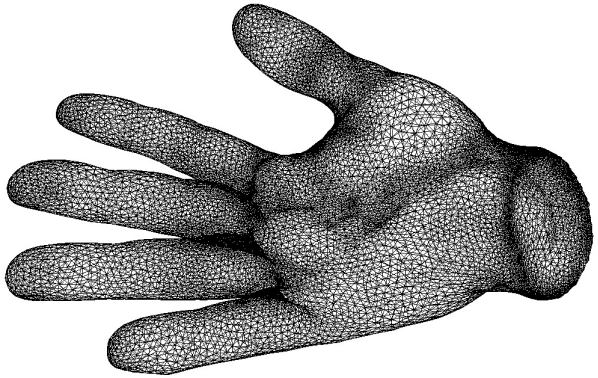
Scaling with Element Number



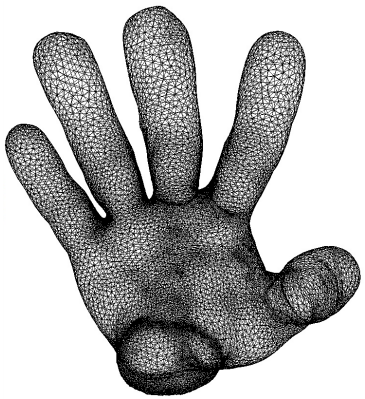
Scaling with Element Order



Application



GPU non-linear neoHookean model.



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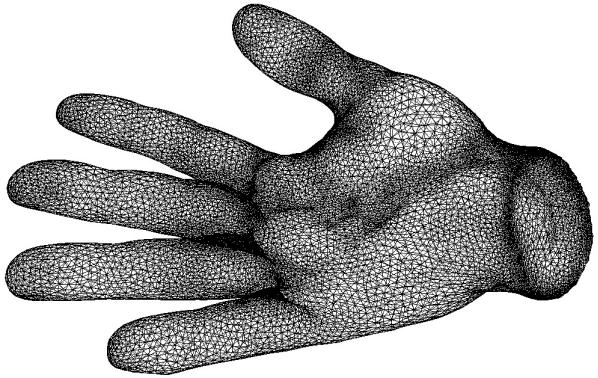

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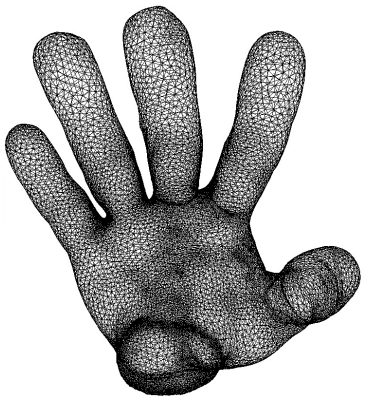

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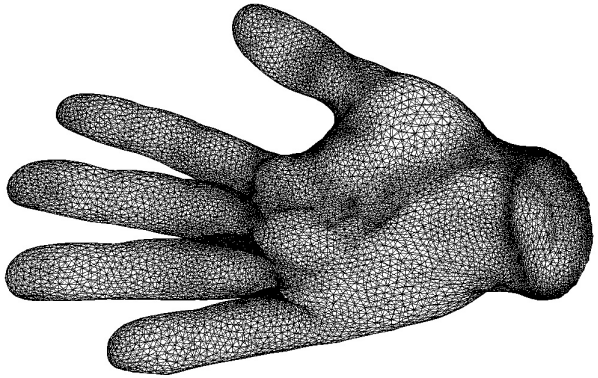

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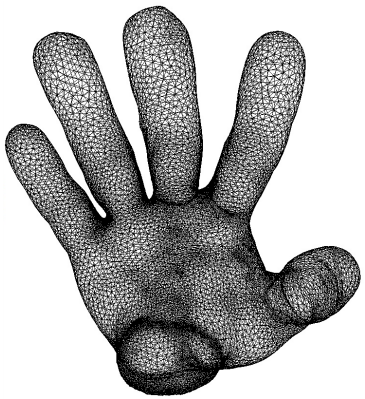
- 28,796 Nodes. 125,127 Elements.



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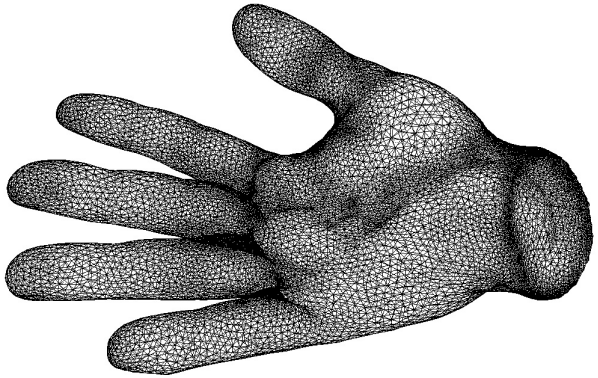

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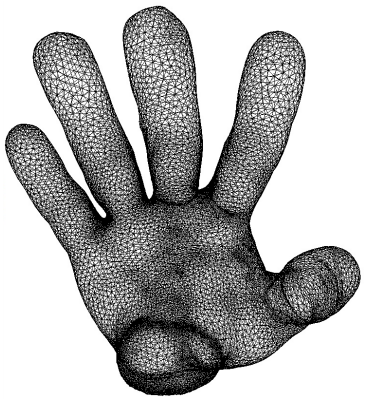

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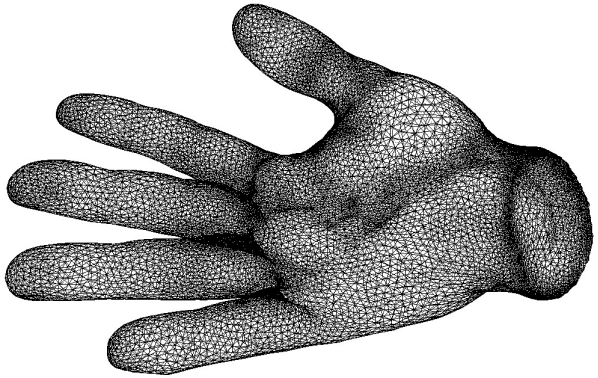

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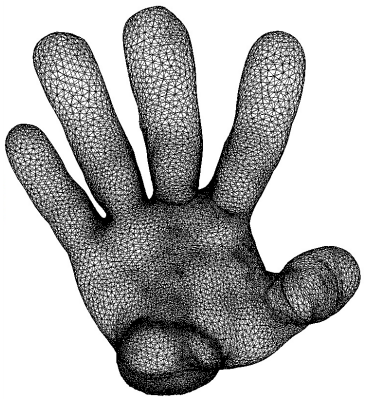

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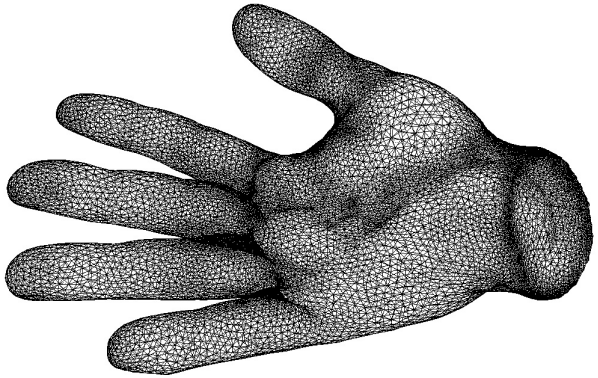

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- Newton-Raphson update at each step.
- Assemble, solve, update, and render at each step.
- 28,796 Nodes. 125,127 Elements.

Preliminary Results:

- Assembly on CPU and transferring requires $\sim$0.5s (optimized).
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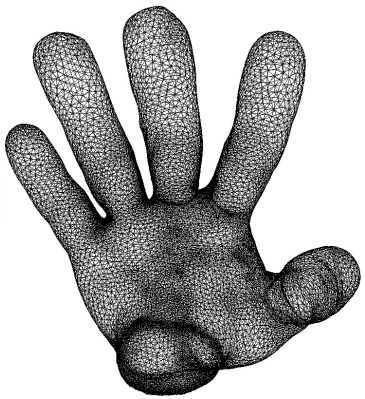

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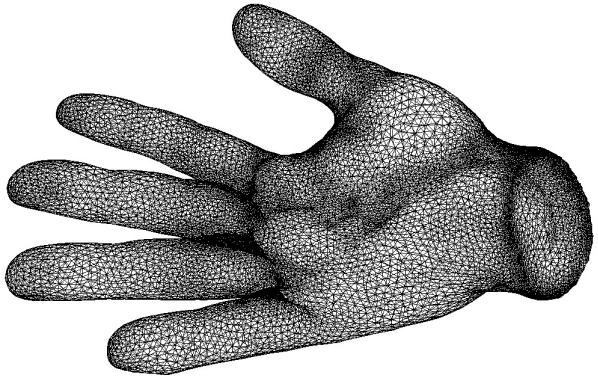

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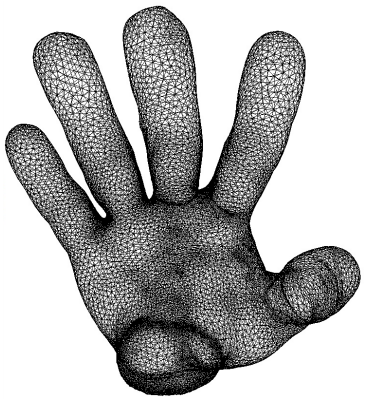

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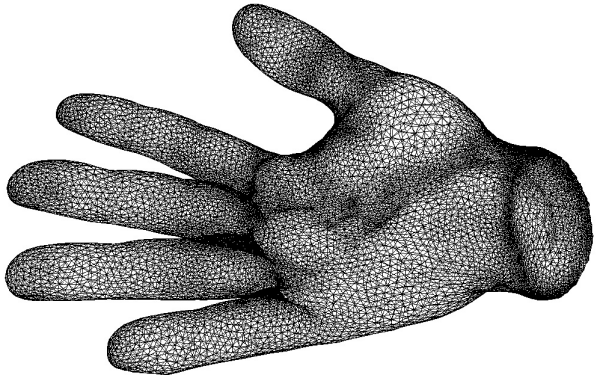

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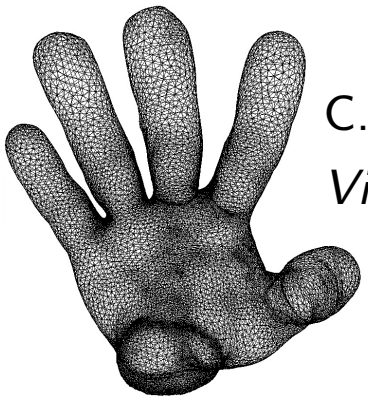

C. Cecka, A. Lew, E. Darve. *Application of Assembly, Solution, and Visualization of FEM on GPUs.* GPU Gems. Preprint.

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- Applying the methods to a high-performance FEM application.